

A general proof of the equivalence between the δN and covariant formalisms

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Recently, the equivalence between the δN and covariant formalisms has been shown, but they assumed Einstein gravity in their proof [arXiv:1201.3163]. They showed that the evolution equation of the curvature covector in the covariant formalism on uniform energy density slicings coincides with that of the curvature perturbation in the δN formalism assuming the coincidence of uniform energy and uniform expansion (Hubble) slicings, which is the case on superhorizon scales in Einstein gravity. In this short note, we explicitly show the equivalence between the δN and covariant formalisms without specifying the slicing condition and the associated slicing coincidence, in other words, regardless of the gravity theory.

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I. INTRODUCTION

Thanks to the current high precision measurements of the Cosmic Microwave Background (CMB), the nature of its temperature anisotropies has been determined to have power spectrum with tiny amplitude ($\sim 10^{-5}$), nearly scale invariance and almost Gaussian statistics [1]. These tiny temperature anisotropies are naturally explained by inflation in the very early universe. Fortunately or unfortunately, there are hundreds or thousands of inflation models consistent with the current observations. As such, possible deviations from Gaussianity of the statistics of the CMB, non-Gaussianities, have recently been attracting the attention of cosmologists as a possible means to constrain inflation models, should they be detected [2]. (See articles in the focus section in CQG [3] and references therein for recent developments.) The PLANCK satellite will indeed provide us with more precise constraints on the statistics of the temperature anisotropies in the next couple of years. To theoretically compute the non-Gaussianity, we need to include the non-linear dynamics of cosmological perturbations.

The simple and straightforward way to handle such non-linear dynamics is the standard perturbative approach [4–7], which can in principle deal with the most general of situations, as long as the perturbation expansion is applicable. However, the equations can become very much involved and quite often the physical transparency may be lost. To resolve this problem, we have two alternative approaches: the δN formalism [8–11] and the covariant formalism [12–16] (see also the review [17]).

The δN formalism corresponds to the leading order approximation of the spatial gradient expansion approach [7, 18–27]. In the gradient expansion approach, the field equations are expanded in powers of spatial gradients and hence it is applicable only to perturbations on very large spatial scales. However, a big advantage is that at leading order in the gradient expansion, which corresponds to the separate universe approach [28], the field equations become ordinary differential equations with respect to time; hence, the physical quantities at each spatial point (where “each point” corresponds to a Hubble horizon size region) evolve in time independently from those in the rest of the space. By solving the fiducial homogeneous equation, we can evaluate the non-linear dynamics of perturbations and evaluate the generated non-Gaussianity.

One of the most important results obtained in the gradient expansion approach is that the full nonlinear curvature perturbation on uniform energy density slices is conserved at leading order in the gradient expansion if the pressure is only a function of the energy density [11] (i.e. the perturbation is purely adiabatic). This is shown only by using the energy conservation law that is satisfied in any gravity theory as long as the theory has general covariance. Thus, without solving the gravitational field equations, one can predict the spectrum and the statistics of the curvature perturbation at horizon re-entry during the late radiation or matter-dominated era once one knows these properties of the curvature perturbation at horizon exit during inflation. (For Galileon or kinetic braiding models [29–40], this does not hold [41, 42])

In the covariant formalism, all quantities are defined in a covariant manner and their evolution equations are also obtained in a fully non-linear and covariant form. Since these quantities behave as tensors, they are much easier to intuitively understand from a geometrical point of view. It is known that the curvature covector is conserved provided the pressure is purely adiabatic. One of the main differences from the gradient expansion approach is that the covariant approach can be applied to the non-linear dynamics at all scales.

However, the relation between quantities in the covariant formalism and those in perturbation theory or the δN formalism is unclear, since all quantities in the covariant formalism are defined in a covariant way. For example, although the correspondence between the curvature covector and the curvature perturbation is revealed perturbatively

[12, 13, 43, 44], it is not well-understood at full non-linear level. The bispectrum of the curvature perturbations in two-field inflation models has been studied in terms of these two different formalism and their coincidence has been shown in [45]. More recently, it has been shown that the evolution equation of the curvature covector on uniform energy density slicings coincides with that of the curvature perturbation in the δN formalism, assuming the coincidence of uniform energy and uniform expansion (Hubble) slicings [46].

In this short note, we give a general proof of the equivalence between the δN and covariant formalisms without specifying the slicing condition and the associated slicing coincidence, in other words, independent of the gravity theory. Here we notice that the δN formalism in linear theory, which was first proposed in [9], gives the final curvature perturbation on the uniform energy density slice. In that paper, they assumed the final time to be some time after complete reheating when the curvature perturbation becomes constant, which means the curvature perturbation in the δN formalism does not depend on the choice of final time, i.e. it is just a constant. As such, in the case of the original δN formalism, there is no evolution equation for the curvature perturbation. However, we do have an evolution equation for the curvature perturbation in the lowest-order gradient expansion. In light of these facts, first, we show that the spatial component of the curvature covector on uniform energy density slicings in the covariant formalism coincides with the spatial gradient of the curvature perturbation on the same slicing as given by the δN formalism. Next, we show that the evolution equation of the curvature covector is equivalent to the evolution equation of the curvature perturbation at the lowest order in the gradient expansion without specifying the slicing condition and slicing coincidence or the gravity theory.

II. BRIEF REVIEWS

A. Gradient expansion approach and δN formalism

In this subsection, we focus on the dynamics on superhorizon scales and use the spatial gradient expansion approach, which will be valid on large scales. We attach a fiducial parameter ϵ to each spatial derivative and expand equations in powers of ϵ . The lowest order in the gradient expansion corresponds to the δN formalism.

We express the metric in the ADM form

$$ds^2 = -\alpha^2 dt + \hat{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt), \quad (2.1)$$

where α , β^i and $\hat{\gamma}_{ij}$ are the lapse function, the shift vector and the spatial metric, respectively. We further decompose the spatial metric as

$$\hat{\gamma}_{ij}(t, x^k) = a^2(t) e^{2\psi(t, x^k)} \gamma_{ij}(t, x^k), \quad \det \gamma_{ij} = 1, \quad (2.2)$$

where $a(t) e^{\psi(t, x^k)}$ is the scale factor at each local point, while $a(t)$ is the scale factor of a fiducial homogeneous universe. By virtue of the separate universe assumption [28], the shift vector is order of ϵ or higher and $\partial_t \gamma_{ij} = \mathcal{O}(\epsilon^2)$ [9, 11]. Then at leading order in gradient expansion, we identify ψ as the nonlinear curvature perturbation [11].

Under the separate universe assumption, the energy-momentum tensor will take the perfect fluid form

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu} + \mathcal{O}(\epsilon^2), \quad (2.3)$$

where $\rho = \rho(t, x^i)$ and $P = P(t, x^i)$ are the energy density and pressure of the fluid. Here we choose the comoving threading as the choice of spatial coordinates

$$v^i = \frac{u^i}{u^0} \left(= \frac{dx^i}{dt} \right) = 0. \quad (2.4)$$

which means the spatial coordinates are chosen so as to comove with the fluid. Then the time component of u^μ is determined by the normalisation condition of u^μ , $u^\mu u_\mu = -1$, as

$$u^0 = \frac{1}{\alpha} + \mathcal{O}(\epsilon^2), \quad (2.5)$$

where we have used the condition $\beta^i = \mathcal{O}(\epsilon)$. The expansion of u^μ is given by

$$\Theta \equiv \nabla_\mu u^\mu = \frac{3}{\alpha} \left(\frac{\partial_t a}{a} + \partial_t \psi \right) + \mathcal{O}(\epsilon^2). \quad (2.6)$$

Next, we write down the local energy conservation equation

$$-u_\mu \nabla_\nu T^{\mu\nu} = u^\mu \nabla_\mu \rho + \Theta(\rho + P) = 0. \quad (2.7)$$

By inserting Eq (2.6), this gives

$$\frac{\partial_t a}{a} + \partial_t \psi = -\frac{\partial_t \rho}{3(\rho + P)} + \mathcal{O}(\epsilon^2). \quad (2.8)$$

In passing, let us choose the uniform energy density slicing. From Eq (2.8), if P is a function of ρ , $P = P(\rho)$, the RHS is independent of spatial coordinates because it is given by a function of ρ , which is a function of time on uniform energy density slices. And hence the curvature perturbation is conserved,

$$\partial_t \psi_E(t, x^i) = \mathcal{O}(\epsilon^2), \quad \text{if } P = P(\rho), \quad (2.9)$$

where the subscript E denotes a quantity evaluated on the uniform energy density slice.

Finally, we define the e -folding number along an integral curve of u^μ :

$$\mathcal{N}(t_F, t_I; x^i) \equiv \frac{1}{3} \int_{t_I}^{t_F} d\tau \Theta(t, x^i), \quad (2.10)$$

where the integral is performed along a constant spatial coordinate line because of the condition of comoving threading Eq (2.4). By rewriting Θ with Eq (2.6), we have

$$\mathcal{N}(t_F, t_I, x^i) = \int_{t_I}^{t_F} dt (H + \partial_t \psi) = N(t_F, t_I) + [\psi(t_F, x^i) - \psi(t_I, x^i)], \quad (2.11)$$

where we have introduced the e -folding number of a fiducial homogeneous universe

$$N(t_F, t_I) \equiv \int_{t_I}^{t_F} dt \frac{\partial_t a(t)}{a(t)} = \log \left[\frac{a(t_F)}{a(t_I)} \right]. \quad (2.12)$$

From Eq (2.11), we see that the difference between the actual e -folding number \mathcal{N} and the background value N gives the change in ψ . Let us consider two slicings, slicings A and B, which coincide at $t = t_I$. The slicing A is such that it starts on a flat slicing at initial time t_I and ends on a uniform energy density slicing at final time t_F . On the other hand, the slicing B is the flat slicing all the time from $t = t_I$ to $t = t_F$. Then, we have

$$\psi_E(t_F, x^i) = \mathcal{N}_A(t_F, t_I; x^i) - N(t_F, t_I) \equiv \delta N(t_F, t_I; x^i). \quad (2.13)$$

This is the non-linear δN formula.

B. Covariant formalism

As we have seen in the last subsection, the δN formalism is a coordinate-based approach focusing on the superhorizon dynamics. On the other hand, in the covariant formalism, all quantities are covariantly defined and this formalism is applicable to all length scales.

First, we define the curvature covector, which is one of the most important quantity in the covariant formalism,

$$\zeta_\mu \equiv \partial_\mu \mathcal{N} - \frac{\dot{\mathcal{N}}}{\dot{\rho}} \partial_\mu \rho. \quad (2.14)$$

where the dot of a scalar quantity denotes the derivative along u^μ , $\dot{\rho} \equiv u^\mu \nabla_\mu \rho$. It is also useful to see the relation between $\dot{\mathcal{N}}$ and Θ from the definition of \mathcal{N} , Eq (2.10),

$$\dot{\mathcal{N}} \equiv u^\mu \nabla_\mu \mathcal{N} = \frac{1}{3} \Theta. \quad (2.15)$$

Here we take the partial derivative of Eq (2.7),

$$\dot{\rho}_\mu + 3\dot{\mathcal{N}} \partial_\mu (\rho + P) + 3\dot{\mathcal{N}}_\mu (\rho + P) = 0, \quad (2.16)$$

where the dot of a vector quantity denotes the Lie derivative of its vector with respect to u^μ ,

$$\dot{V}_\mu \equiv \mathcal{L}_u V_\mu \equiv u^\nu \partial_\nu V_\mu + V_\nu \partial_\mu u^\nu. \quad (2.17)$$

After some calculation, we can rewrite Eq (2.16) as,

$$\dot{\zeta}_\mu = -\frac{\Theta}{3(\rho + P)} \left(\partial_\mu P - \frac{\dot{P}}{\dot{\rho}} \partial_\mu \rho \right). \quad (2.18)$$

If the pressure is given by a function of the energy density, $P = P(\rho)$, the right hand side apparently vanishes and the curvature covector is conserved.

Before closing this subsection, we leave a brief comment on the initial condition of the curvature covector. In the covariant formalism, the e -folding number is defined as the integration of the expansion along an integral curve of u^μ as in Eq (2.10). Then, the derivative of \mathcal{N} with respect to u^μ , Eq (2.15), is well-defined and gives the expansion. On the other hand, how can we evaluate the partial derivative of the e -folding number, $\partial_\mu \mathcal{N}$, in Eq (2.14)? Since we do not specify the initial time of the integration, the e -folding numbers for each integral curve cannot be directly compared. This means the curvature covector has an arbitrariness in the choice of the initial hypersurface, which we thus need to specify before being able to discuss the relation with the curvature perturbation in the δN formalism, for example. Hereinafter, we introduce a prefix for ζ_μ to make clear the dependence of the curvature covector on the initial slicing. In particular, ${}_G\zeta_\mu$ and ${}_F\zeta_\mu$ are defined as the curvature covector on the general initial slice and the initial flat slice respectively,

$${}_G\zeta_\mu \equiv \zeta_\mu \text{ on the general initial slice}, \quad (2.19)$$

$${}_F\zeta_\mu \equiv \zeta_\mu \text{ on the initial flat slice}. \quad (2.20)$$

III. EQUIVALENCES

A. δN formula and the curvature covector

In this subsection, the relation between the curvature covector and the curvature perturbation in the δN formalism is revealed. In particular, we show that the spatial component of the curvature covector exactly coincides with the spatial derivative of the curvature perturbation on the uniform energy density slicing given by the δN formalism when the initial conditions for the curvature covector are given on an appropriate slicing, i.e. flat slicing. As we have seen in the last subsection, we need to specify the initial hypersurface for ζ_μ to compare with the curvature perturbation. Of course, since the initial hypersurface for ζ_μ can be chosen arbitrarily, the spatial component of the curvature covector does not, in general, coincide with the spatial derivative of the curvature perturbation in the δN formalism. This is because the initial hypersurface in the δN formalism is chosen to be a flat one.

Now we show the equivalence between the δN formula and the curvature covector. From Eq (2.14), the spatial component of the curvature covector on the uniform energy density slicing is given by the spatial derivative of the e -folding number, which corresponds to the difference in the curvature perturbation on initial and final slices

$${}_G\zeta_i|_E(t, x^j) = \partial_i \mathcal{N}_E(t, x^j) = \partial_i [\psi_E(t, x^j) - \psi(t_I, x^j)], \quad (3.1)$$

where ψ_E means the curvature perturbation on uniform energy density slices. By appropriately choosing the initial slicing to be a flat slicing, we can further rewrite the above equation as

$${}_F\zeta_i|_E(t, x^j) = \partial_i \psi_E(t, x^j) = \partial_i [\delta N(t, t_I; x^j)]. \quad (3.2)$$

This apparently shows the equivalence between the δN formula Eq (2.13) and the curvature covector. However, if the initial hypersurface is not chosen to be a flat one, the equivalence does not hold.

B. Lowest order gradient expansion and the covariant formalism

Next, we show the equivalence between the lowest order gradient expansion and the covariant formalism without specifying the time slicing at final time, that is, in a gauge invariant way.

First, the time component of curvature covector is expressed as

$${}_G\zeta_0 = \partial_t \mathcal{N} + \frac{\partial_t \rho}{3(\rho + P)} = \mathcal{O}(\epsilon^2), \quad (3.3)$$

and the spatial component is

$${}_G\zeta_i = \partial_i \mathcal{N} + \frac{\partial_i \rho}{3(\rho + P)} = \partial_i \left(\int dt \partial_t \psi \right) + \frac{\partial_i \rho}{3(\rho + P)}. \quad (3.4)$$

The evolution equation for the spatial component of the curvature covector is given by the spatial part of Eq (2.18)

$${}_G\dot{\zeta}_i = -\frac{\Theta}{3(\rho + P)} \left(\partial_i P - \frac{\dot{P}}{\rho} \partial_i \rho \right) = \frac{1}{\alpha} \frac{\partial_t \rho}{3(\rho + P)^2} \left(\partial_i P - \frac{\partial_t P}{\partial_t \rho} \partial_i \rho \right), \quad (3.5)$$

where the explicit form of the LHS is given by

$${}_G\dot{\zeta}_i = u^\mu \partial_\mu ({}_G\zeta_i) + {}_G\zeta_\mu \partial_i u^\mu = u^0 \partial_t ({}_G\zeta_i) + {}_G\zeta_0 \partial_i u^0 = \frac{1}{\alpha} \left[\partial_t ({}_G\zeta_i) - {}_G\zeta_0 \frac{\partial_t \alpha}{\alpha} \right]. \quad (3.6)$$

By inserting Eq (3.4), this reduces to

$${}_G\dot{\zeta}_i = \frac{1}{\alpha} \partial_t ({}_G\zeta_i) + \mathcal{O}(\epsilon^2) = \frac{1}{\alpha} \left[\partial_t (\partial_i \psi) + \frac{\partial_t \partial_i \rho}{3(\rho + P)} - \frac{\partial_i \rho \partial_t (\rho + P)}{3(\rho + P)^2} \right] + \mathcal{O}(\epsilon^2), \quad (3.7)$$

where we have used Eq (3.3) in the first equality. Finally, Eq (3.5) is rewritten as

$$\begin{aligned} \partial_i (\partial_t \psi) &= \frac{\partial_t \rho}{3(\rho + P)^2} \left(\partial_i P - \frac{\partial_t P}{\partial_t \rho} \partial_i \rho \right) - \frac{\partial_t \partial_i \rho}{3(\rho + P)} + \frac{\partial_i \rho \partial_t (\rho + P)}{3(\rho + P)^2} \\ &= -\partial_i \left[\frac{\partial_t \rho}{3(\rho + P)} \right]. \end{aligned} \quad (3.8)$$

By integrating Eq (3.8) over x^i , we have

$$\partial_t \psi + C(t) = -\frac{\partial_t \rho}{3(\rho + P)}, \quad (3.9)$$

where $C(t)$ is a integration constant. By appropriately choosing $C(t)$ so that it coincides with $\partial_t a/a$, we can see the coincidence of Eq (3.9) with Eq (2.8). This shows the equivalence between the lowest order gradient expansion and the covariant formalism.

IV. CONCLUSION

In this short note, we have given a general proof of the equivalence between the δN and covariant formalisms without specifying the slicing condition and the associated slicing coincidence, in other words, regardless of the gravity theory. First, we have shown that the spatial component of the curvature covector on uniform slicings coincides with the spatial gradient of the curvature perturbation on the same slice which is given by the δN formalism when the initial hypersurface for the curvature covector is appropriately chosen so as to be a flat slicing. Next, we have shown that the evolution equation of the curvature covector is equivalent to the evolution equation of the curvature perturbation at the lowest order in the gradient expansion without specifying a slicing condition and slicing coincidence or the gravity theory.

In passing we have noted that there is an implicit initial slice dependence in the covariant formalism. Therefore we have emphasised that the equivalence can be shown only after the initial slice has been clearly specified. (In other words, ζ_μ is not gauge-invariant unless the initial slice has been clearly specified.)

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